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Conditional generation of an arbitrary superposition of coherent states

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We present a scheme to conditionally generate an arbitrary superposition of a pair of coherent states from a squeezed vacuum by means of the modified photon subtraction where a coherent state ancilla and two on/off type detectors are used. We show that, even including realistic imperfections of the detectors, our scheme can generate a target state with a high fidelity. The amplitude of the generated states can be amplified by conditional homodyne detections.

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I. INTRODUCTION

Conditional quantum operation based on photon detection plays an important role in recent optical quantum information processing. Particularly, in the continuous variable regime, it is the only available tool with current technology to generate non-Gaussian states from Gaussian ones. A typical example is the 'photon subtraction' operation, in which a nonclassical input state is split by a highly transmissive beamsplitter (BS) and the reflected state is measured by a photon number resolving detector (PNRD). Selecting the event that the detector observes photons, one obtains a non-Gaussian transformation from the input to output quantum state.

This type of conditional operation was formulated in [1]. Dakna *et al.* [2] then showed that applying it to a squeezed vacuum, one can generate a non-Gaussian state which is close to the superposition of coherent states with plus or minus phase

$$|C_{\pm}(\alpha)\rangle = \frac{1}{\sqrt{\mathcal{N}_{\pm}}}\left(|\alpha\rangle \pm |-\alpha\rangle\right), \qquad (1)$$

with very high fidelity, where $|\alpha\rangle$ is a coherent state with the amplitude of α and $\mathcal{N}_{\pm}$ is the normalization factor.

Recently, such state has been experimentally generated by a single photon subtraction from pulsed [3, 4] and CW [5, 6] squeezed vacua. In these experiments, since a reflected beam includes sufficiently small average number of photons ($\bar{n} \ll 1$), single photon detection was approximately realized by use of an avalanche photodiode (APD) which is often called as 'on/off' type detector since it discriminates only a presence of photons instead of resolving photon numbers.

The progress of these experiments promises the realizations of more complicated applications of photon subtraction proposed so far, including the improvement of quantum teleportation [7, 8, 9] and entanglement-assisted coding [10], entanglement distillation [11], loophole free tests of Bell's inequalities [12, 13, 14], and optical quantum computations in quadrature basis [15, 16] or superposed coherent state basis [17, 18]. In the last application, $|C_{\pm}(\alpha)\rangle$ with appropriate α is required as an ancillary state. To prepare such ancillae, the method to conditionally amplify α with on/off detectors has been proposed [19, 20]. Fiurášek *et al.* [21] also recently showed that one can arbitrarily synthesize a single-mode quantum state up to the N-photon eigenstate by concatenating squeezing operations and N times single photon subtractions [21].

In this paper, we propose a method to conditionally generate the state in which two coherent states are superposed with *arbitrary* ratio and phase, $c_+|\alpha\rangle + c_-|-\alpha\rangle$. This is accomplished by a simple modification of the scheme proposed by Dakna *et al.* (DAOKW) [2]. We first discuss an ideal scheme using two PNRDs and a qubit ancilla, which produces a superposition of the one- and two-photon subtracted states. We show that such state fairly well approximates the target state $c_+|\alpha\rangle + c_-|-\alpha\rangle$. We next present a more practical scheme where PNRDs and a qubit ancilla are replaced by the on/off detectors and a coherent state ancilla. Even including practical imperfections of the detectors, it can generate the target state with a high fidelity.

Our scheme should be compared with the one by Fiurášek *et al.* [21], which requires N detectors to synthesize a state consisting of the number states up to $|N\rangle$. Ours, on the other hand, uses only two detectors to synthesize a fully continuous variable state while, in return, the class of states to be generated is restricted.

Finally, we show that our scheme is useful to simplify the setup of the conditional amplification of the superpositions of coherent states originally proposed in [19, 20].

The paper is organized as follows. In Sec. II, we discuss an ideal setup with PNRDs and a qubit ancilla and how the scheme's parameters are optimized to generate desired superposed states. In Sec. III, a practical scheme using on/off detectors and a coherent state ancilla is shown and its experimental feasibility is numerically examined. An application of our scheme to the conditional amplification of superposed coherent states is shown in Sec. IV and Sec. V concludes the paper.

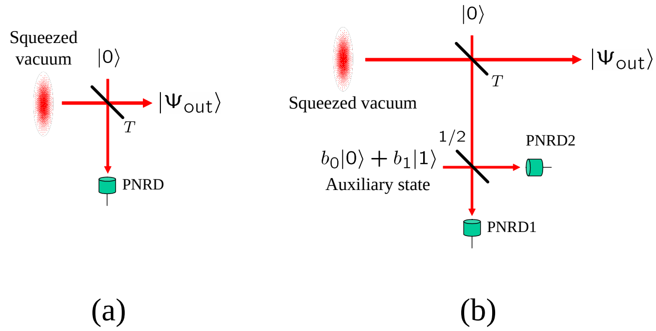

FIG. 1: (a) Generation of a plus- or minus-superposition of coherent states via photon subtraction operation with a photon number resolving detector (PNRD). (b) Generation of an arbitrary superposition of coherent states with PNRDs and a qubit ancilla.

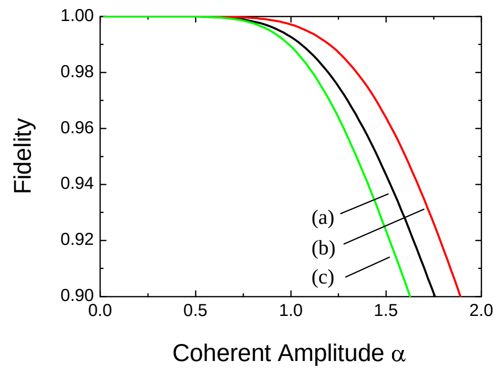

FIG. 2: Fidelities between the photon subtracted states and the ideal superposition of coherent states. (a) $|\langle\alpha|\phi_+\rangle|^2$, (b) $|\langle C_-(\alpha)|\Psi_1\rangle|^2$, and (c) $|\langle C_+(\alpha)|\Psi_2\rangle|^2$.

II. GENERATION OF AN ARBITRARILY SUPERPOSITION OF COHERENT STATES

Figure 1(a) illustrates the DAOKW photon subtraction scheme [2]. A squeezed vacuum with the squeezing parameter r is mixed with a vacuum by a highly transmissive BS and the reflected part of the state is detected by a PNRD. When the reflected part is projected onto the photon number eigenstate $|m\rangle$ ($m > 0$), the state remained in the transmitted mode is reduced to be the m photon subtracted squeezed vacuum state that can be described by a minus- or plus-superposition of two distinct states as

$$|\Psi_m\rangle = A(|\Psi_m^{(+)}\rangle + (-1)^m|\Psi_m^{(-)}\rangle), \qquad (2)$$

where A is the normalization factor [22]. It was shown that, with an appropriate input squeezed vacuum, the states $|\Psi_m^{(\pm)}\rangle$ are very close to the coherent states $|\pm\alpha_m\rangle$ and thus the states $|\Psi_m\rangle$ are also very close to a superposition of coherent states.

Let us extend the above scheme as illustrated in Fig. 1(b). Let the upper BS has the power transmittance $T \approx 1$ and the lower be a balanced BS. The reflected part of the state is mixed with the auxiliary state $b_0|0\rangle + b_1|1\rangle$ and then each port is incident into a PNRD. After some calculations, one finds that if the measurement outcome of the two detectors is $(2, 0)$ or $(0, 2)$, the reflected part is effectively projected onto $\mp b_1^*/\sqrt{2}|1\rangle + b_0^*/\sqrt{2}|2\rangle$ and the transmitted state conditioned on either of these outcomes has the form

$$|\Psi_{\rm out}\rangle = a_1|\Psi_1\rangle + a_2|\Psi_2\rangle, \qquad (3)$$

where a_1 and a_2 are the functions of b_0, b_1, and T. Since $|\Psi_m\rangle$ can be regarded as a superposition specified in Eq. (1), the state $|\Psi_{\rm out}\rangle$ is also expected to be a superpostion of $|\pm\alpha\rangle$ with the controlled ratio and phase by choosing ancilla parameters b_0 and b_1 appropriately.

Now let us see the state in Eq. (3) more carefully. To show $|\Psi_{\rm out}\rangle$ to be a superposition of two (classical) macroscopically distinct states, one has to find the following decompositions,

$$|\Psi_1\rangle = \frac{1}{2c_1}\left(|\phi_+\rangle - |\phi_-\rangle\right), \qquad (4)$$

$$|\Psi_2\rangle = \frac{1}{2c_2}\left(|\phi_+\rangle + |\phi_-\rangle\right), \qquad (5)$$

in which $|\phi_\pm\rangle$ are close enough to the coherent states $|\pm\alpha\rangle$. c_1 and c_2 are the normalization factors satisfying $|\phi_\pm\rangle = c_2|\Psi_2\rangle \pm c_1|\Psi_1\rangle$. Note that the decomposition described in Eq. (2) [2] is not optimal in our purpose since $|\Psi_1\rangle$ and $|\Psi_2\rangle$ do not share the common decomposed components.

The optimal $|\phi_\pm\rangle$ to maximize the fidelity $|\langle\phi_\pm|\pm\alpha\rangle|^2$ can be derived from the exact expression of $|\Psi_m\rangle$ [22] and Eqs. (4) and (5) as

$$c_1 = \sqrt{\frac{3\lambda T}{(1+\lambda T)(1+2\lambda T)}}, \qquad (6)$$

$$c_2 = \sqrt{\frac{1+2\lambda^2T^2}{(1+\lambda T)(1+2\lambda T)}}, \qquad (7)$$

where $\lambda = \tanh r$ is the squeezing parameter and the amplitude of the corresponding coherent states is given by

$$|\pm\alpha\rangle = \left|\pm\sqrt{\frac{3\lambda T}{1-\lambda^2T^2}}\right\rangle. \qquad (8)$$

Then we have quasi-coherent states

$$|\phi_\pm\rangle = \frac{(1-\lambda^2T^2)^{3/4}}{2\sqrt{(1+\lambda T)(1+2\lambda T)}}\sum_{n=0}^{\infty}\frac{(2n+2)!}{(n+1)!}\left(\frac{\lambda T}{2}\right)^n \left(\frac{1-\lambda^2T^2}{\sqrt{(2n)!}}|2n\rangle \pm \sqrt{\frac{3\lambda T}{(2n+1)!}}|2n+1\rangle\right), \qquad (9)$$

and the fidelity between Eqs. (8) and (9) is given by

$$\begin{aligned} F &= |\langle\alpha|\phi_+\rangle|^2 \\ &= \sqrt{1-\lambda^2T^2}(1+\lambda T)(1+2\lambda T)\exp\left[-\frac{3\lambda T}{1+\lambda T}\right], \end{aligned} \tag{10}$$

which is plotted in Fig. (2) by the black line (line (a)). For $\alpha < 1$, more than 0.99 fidelity is achieved. The validity of this optimization is also confirmed by looking at the fidelities $|\langle\Psi_1|C_-(\alpha)\rangle|^2$ and $|\langle\Psi_2|C_+(\alpha)\rangle|^2$ for the same α. These are plotted in the same figure by the red (line (b)) and green (line (c)) lines, respectively.

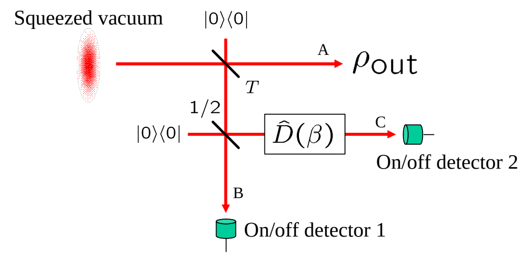

FIG. 3: A schematic of the generation of an aribitrary superposition of coherent states with on/off detectors and a displacement operation.

III. PRACTICAL SETUP WITH ON/OFF DETECTORS

Preparing PNRDs and a photon number qubit ancilla is still somehow challenging with current technology. In this section, we show a modified and more practical scheme in which PNRDs and a qubit ancillary state are replaced with on/off detectors and a coherent state, respectively.

The modified scheme is depicted in Fig. 3. The reflected state from the first BS with the transmittance T is further split by the second balanced BS. One beam is directly measured by an on/off detector (mode B) and the other is first shifted by the displacement operator $\hat{D}(\beta) = \exp[\beta\hat{a}^\dagger - \beta^*\hat{a}]$ and then measured by another on/off detector (mode C). It is well known that a displacement operation is realized by interfering the signal with an auxiliary coherent state $|\beta/\sqrt{1-T_D}\rangle$ by a BS with the transmittance T_D. In the limit of $T_D \to 1$, this operation is exactly the same as $\hat{D}(\beta)$. The output state is conditionally selected only when both detectors are simultaneously clicked by photons. The photons detected in mode B always come from the squeezed vacuum while, in mode C, the photons from the squeezed vacuum are interfered by the displacement. This quantum interference and the on/off detection realizes a projection onto a superposition of different photon number states.

The positive operator-valued measure (POVM) for on/off detectors is described by $\{\hat{\Pi}_{\rm off}, \hat{\Pi}_{\rm on}\}$ where $\hat{\Pi}_{\rm off} = |0\rangle\langle 0|$ and $\hat{\Pi}_{\rm on} = \hat{I} - \hat{\Pi}_{\rm off}$ and $\hat{I}$ is an identity operator. Similarly, when a displacement operation $\hat{D}(\beta)$ is placed before the detector, as in mode C, the total on/off POVM is expressed as

$$\hat{\Pi}_{\rm off}(\beta) = \hat{D}^\dagger(\beta)|0\rangle\langle 0|\hat{D}(\beta) = |-\beta\rangle\langle -\beta|, \tag{11}$$

$$\begin{aligned} \hat{\Pi}_{\rm on}(\beta) &= \hat{D}^\dagger(\beta)(\hat{I} - |0\rangle\langle 0|)\hat{D}(\beta) \\ &= \hat{I} - |-\beta\rangle\langle -\beta|. \end{aligned} \tag{12}$$

The average photon number reflected to mode B from the initial squeezed vacuum is given by $(1-T)\sinh^2 r$. For moderate squeezing, this is sufficiently small to assume that the reflected beam in mode B contains maximally one photon and to ignore the more than one photon eigenspace at the measurement process (e.g. $(1-T)\sinh^2 r \sim 0.005$ for $r = 0.3$ and $T = 0.95$). When $|\beta|^2$ in Eqs. (11) and (12) is also sufficiently small such that one can approximate as $|-\beta\rangle \approx |0\rangle - \beta|1\rangle$, $\hat{\Pi}_{\rm off}(\beta)$ in mode C acts as the projection onto $|0\rangle - \beta|1\rangle$, and $\hat{\Pi}_{\rm on}(\beta)$ as the projection onto the orthogonal superposition $\beta^*|0\rangle + |1\rangle$. As a consequence, when both detectors are clicked, the reflected part of the state is projected onto

$$\begin{aligned} & {}_C\langle 0|\hat{B}^\dagger_{1/2}|1\rangle_B\,(\beta^*|0\rangle_C + |1\rangle_C) \\ & \propto {}_C\langle 0|\left\{-\beta^*(|01\rangle - |10\rangle) - (|02\rangle - |20\rangle)\right\}_{BC} \\ & = \beta^*|1\rangle_B + |2\rangle_B, \end{aligned} \tag{13}$$

where the normalization factors and global phases are omitted and $\hat{B}_T = \exp[\theta(\hat{a}^\dagger\hat{b} - \hat{a}\hat{b}^\dagger)]$ is a BS operator with $\cos\theta = \sqrt{T}$.

On the other hand, the state after the first BS is described as

$$\begin{aligned} \hat{B}_T\left(\hat{S}(r)|0\rangle_A\right)|0\rangle_B &= \sqrt{\mathcal{P}_0}|\Psi_0\rangle_A|0\rangle_B + \sqrt{\mathcal{P}_1}|\Psi_1\rangle_A|1\rangle_B \\ &\quad + \sqrt{\mathcal{P}_2}|\Psi_2\rangle_A|2\rangle_B + \cdots \end{aligned} \tag{14}$$

where $S(r) = \exp[r/2(\hat{a}^2 - \hat{a}^{\dagger\,2})]$ is a squeezing operator and $\mathcal{P}_m$ is the probability to observe m photons in modes B and C [2],

$$\begin{aligned} \mathcal{P}_m &= \sqrt{\frac{1-\lambda^2}{1-\lambda^2T^2}}\left[\frac{\lambda^2T^2(1-T)}{T(1-\lambda^2T^2)}\right]^m \\ &\quad \times \sum_{k=0}^{[m/2]} \frac{m!}{(m-2k)!(k!)^2(2\lambda T)^{2k}}. \end{aligned} \tag{15}$$

From Eqs. (13) and (14), we have the conditional output for the simultaneous click as

$$|\Psi_{\rm out}\rangle \propto \beta\sqrt{\mathcal{P}_1}|\Psi_1\rangle + \sqrt{\mathcal{P}_2}|\Psi_2\rangle. \tag{16}$$

Consequently, for the generation of the superposition state $c_+|\phi_+\rangle + c_-|\phi_-\rangle$, the optimal displacement β is derived from Eqs. (4–7), (15), and (16) and given as

$$\beta = \frac{c_+ - c_-}{c_+ + c_-}\left(\frac{3\lambda(1-T)}{2(1-\lambda^2T^2)}\right)^{1/2}, \tag{17}$$

which is valid under the condition of $|\beta|^2 \ll 1$, i.e.

$$\left|\frac{c_+ - c_-}{c_+ + c_-}\right|^2 \ll \frac{2(1-\lambda^2 T^2)}{3\lambda(1-T)}. \quad (18)$$

Note that Eq. (17) is almost optimal for arbitrary β although this condition will be broken when $c_+ + c_- \sim 0$ i.e. one wants to generate $|\Psi_{\text{out}}\rangle \sim |\Psi_1\rangle$. For large $|\beta|^2$, Eq. (12) up to one photon state is given by

$$\hat{\Pi}_{\text{on}}(\beta) \to (1-e^{-|\beta|^2})\hat{I} + e^{-|\beta|^2}(\beta^*|0\rangle + |1\rangle)(\beta\langle 0| + \langle 1|). \quad (19)$$

Although it makes the output as a mixed state of

$$\hat{\rho}_{\text{out}} = (1-e^{-|\beta|^2})|\Psi_1\rangle\langle\Psi_1| + e^{-|\beta|^2}|\Psi_{\text{out}}\rangle\langle\Psi_{\text{out}}|, \quad (20)$$

this is clearly a negligible error since $|\langle\Psi_{\text{out}}|\Psi_1\rangle|^2$ exponentially approaches to unit.

In the rest of this section, we numerically examine the conditional outputs under realistic conditions. In practice, there is always finite probability to detect more than one photon at each detector. Moreover, the detectors themselves have finite imperfections. The POVM for an imperfect on/off detector with the displacement operation $\hat{D}(\alpha)$ is given by

$$\hat{\Pi}_{\text{off}}(\alpha,\eta,\nu) = e^{-\nu}\sum_{m=0}^{\infty}(1-\eta)^m \hat{D}^\dagger(\alpha)|m\rangle\langle m|\hat{D}(\alpha), \quad (21)$$

$$\hat{\Pi}_{\text{on}}(\alpha,\eta,\nu) = \hat{I} - \hat{\Pi}_{\text{off}}(\alpha), \quad (22)$$

where η and ν are the quantum efficiency and dark count of the detector, respectively.

This kind of detectors makes the output an unwanted mixed state. To derive photon subtracted states under these conditions, it is useful to use the characteristic functions to describe the states and POVMs [23, 24, 25]. Since the input squeezed vacuum is a Gaussian state, its characteristic function can be described as

$$\chi_{\text{SV}}(\omega) = \exp\left[-\frac{1}{4}\omega^T\Gamma_{\text{SV}}\omega\right], \quad (23)$$

where $\omega = (u,v)^T$ is a two dimensional vector and Γ_{SV} is the covariance matrix for the squeezed vacuum

$$\Gamma_{\text{SV}} = \begin{bmatrix} e^{2r} & 0 \\ 0 & e^{-2r} \end{bmatrix}. \quad (24)$$

The mixing of a squeezed vacuum and a vacuum by a BS is described by a linear transformation

$$\Gamma_{\text{SV}} \oplus \Gamma_{\text{vac}} \to S_{BS}^T(T)\Gamma_{\text{SV}} \oplus \Gamma_{\text{vac}} S_{BS}(T), \quad (25)$$

where $\Gamma_{\text{vac}} = \mathbf{I}$ is the covariance matrix for the vacuum state and $S_{BS}(T)$ is the 4×4 matrix

$$S_{BS}(T) = \begin{pmatrix} \sqrt{T}\,\mathbf{I} & \sqrt{1-T}\,\mathbf{I} \\ -\sqrt{1-T}\,\mathbf{I} & \sqrt{T}\,\mathbf{I} \end{pmatrix}. \quad (26)$$

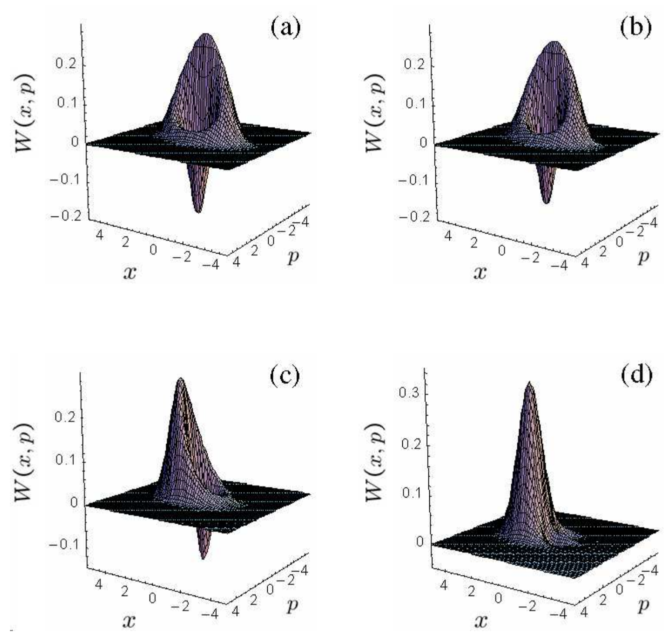

FIG. 4: The Wigner functions of the superpositions of coherent states $(c_+|\phi_+\rangle + c_-|\phi_-\rangle)/\mathcal{N}$ generated by the on/off detector photon subtractions illustrated in Fig. (3) for (a), (b) $\{c_+, c_-\} = \{1, i\}$, (c) $\{c_+, c_-\} = \{3, -1\}$, and (d) $\{c_+, c_-\} = \{1, 0\}$ where (a) $r = 0.3$, $T = 0.999$, $\alpha = 0.97$ with the ideal detectors of $\eta = 1$, and $\nu = 0$, and (b), (c), (d) $r = 0.3$, $T = 0.95$, $\alpha = 0.95$, with the practical imperfection of detectors, $\eta = 0.1$, and $\nu = 10^{-7}$. The fidelities between the plotted state and the ideal state $(c_+|\alpha\rangle + c_-|-\alpha\rangle)/\mathcal{N}$ are (a) $F = 0.993$, (b) $F = 0.952$, (c) $F = 0.978$, and (d) $F = 0.994$.

Then the covariance matrix after the two BSs in Fig. 3 is given by

$$\tilde{\Gamma} = (I \oplus S_{BS}^T(1/2))(S_{BS}^T(T) \oplus I)(\Gamma_{\text{SV}} \oplus \Gamma_{\text{vac}} \oplus \Gamma_{\text{vac}}) \times (S_{BS}(T) \oplus I)(I \oplus S_{BS}(1/2)). \quad (27)$$

Let the characteristic function corresponding to $\tilde{\Gamma}$ be $\chi_{\text{in}}(\omega_{\mathbf{A}}, \omega_{\mathbf{B}}, \omega_{\mathbf{C}})$. The output characteristic function conditioned on the simultaneous click in both two detectors is then given by

$$\chi_{\text{out}}(\omega_{\mathbf{A}}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_{\mathbf{B}} d\omega_{\mathbf{C}}\, \chi_{\text{in}}(\omega_{\mathbf{A}}, \omega_{\mathbf{B}}, \omega_{\mathbf{C}}) \times \chi_{\text{on}}(-\omega_{\mathbf{B}}, -\omega_{\mathbf{C}}), \quad (28)$$

where

$$\chi_{\text{on}}(\omega_{\mathbf{B}}, \omega_{\mathbf{C}}) = \chi_{\text{on}}(0, \omega_{\mathbf{B}})\chi_{\text{on}}(-\beta^*, \omega_{\mathbf{C}}), \quad (29)$$

and $\chi_{\text{on}}(\alpha, \omega)$ corresponds to the characteristic function for the POVM defined in Eq. (22). Finally, the Wigner function for the output is given by the Fourier transform of the characteristic function as

$$W_{\text{out}}(\mathbf{z}) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\omega\, \chi_{\text{out}}(\omega)\exp\left[-i\omega^T\mathbf{z}\right], \quad (30)$$

where $\mathbf{z} = (x, p)^T$.

Figure 4(a) shows $W_{\text{out}}(\mathbf{z})$ corresponding to the state $|\phi_+\rangle + i|\phi_-\rangle$ with nearly ideal parameters, $T = 0.999$, unit quantum efficiency, and zero dark counts. The squeezing parameter $r = 0.3$ corresponds to the coherent amplitude of $\alpha = 0.97$. The fidelity between this output and $|\alpha\rangle + i|-\alpha\rangle$ is $F = 0.993$. Note that this is the same as the state generated from a coherent state by strong Kerr nonlinear evolution [26]. Figures 4(b)-(d) plot the Wigner functions with realistic detectors ($\eta = 0.1$ and $\nu = 10^{-7}$) for different $\{c_+, c_-\}$. Even with such imperfect detectors, high fidelities (> 0.95) could be achieved.

IV. AMPLIFICATION OF THE SUPERPOSED STATES VIA CONDITIONAL HOMODYNE DETECTION

The fidelity between the state generated from our scheme and the ideal superposition of coherent states starts to decrease rapidly for $\alpha > 1$. To realize surely macroscopic superposition or to apply these states to the coherent state superposition based quantum computation scheme [17], the superposed states are required to have larger amplitudes. One approach for the production of such a state is to introduce PNRDs in our scheme (Fig. 3) to generate a superposition of $|\Psi_m\rangle$ and $|\Psi_{m+1}\rangle$ for large m. The other approach may be to apply the conditional amplification process proposed in [19], which does not require PNRDs. It was shown that if one can prepare two inputs $|\alpha\rangle + e^{i\varphi}|-\alpha\rangle$ and $|\beta\rangle + e^{i\phi}|-\beta\rangle$, they can be conditionally transformed to the state $|\gamma\rangle + e^{i(\varphi+\phi)}|-\gamma\rangle$, where $\gamma = \sqrt{\alpha^2+\beta^2}$, by using BSs, an auxiliary coherent state, and two on/off detectors. It could be used to amplify the initial state of $S(r)|1\rangle$, which well approximates $|C_-(\alpha)\rangle$ for $|\alpha|^2 \le 1$, and the scalability of the repetitive amplification process was discussed in detail in [20].

Application of our scenario to their scheme allows us to generate $|\gamma\rangle + e^{i\varphi}|-\gamma\rangle$ with large γ and arbitrary φ. Moreover, we will show in this section that the two on/off detectors used in the scheme of Ref. [19, 20] can be simply replaced by a homodyne detector although the latter acts as a Gaussian operation. Note that it is not prohibited to transform non-Gaussian states to the other non-Gaussian states by only Gaussian operations. The conditional homodyne detection technique has been applied to purify the coherent state superpositions [27] or the squeezed states suffering non-Gaussian noises [28].

The schematic of the conditional amplification via homodyne detection is shown in Fig. 5(a). To explain how it works, let us see a simple example where we have $|C_+(\alpha)\rangle$ and $|C_-(\alpha)\rangle$ as two input states. These states are combined by a balanced BS as

$$(|\alpha\rangle + |-\alpha\rangle)(|\alpha\rangle - |-\alpha\rangle) \xrightarrow{\text{BS}} (|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle)|0\rangle + |0\rangle(|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle), \tag{31}$$

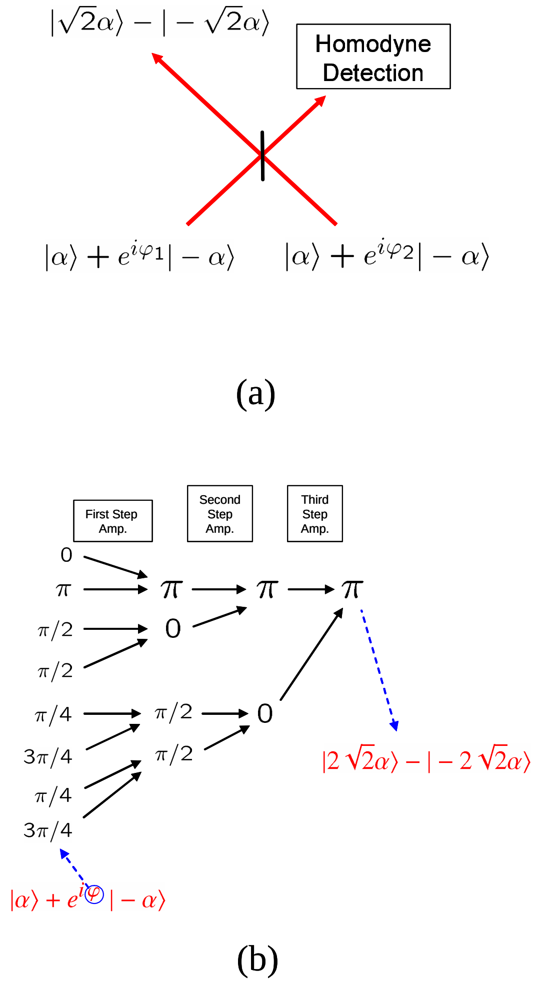

FIG. 5: (a) A schematic of the conditional amplification of the coherent state superpositions with homodyne detection. (b) The three-step concatenation of the amplification process. The 8 initial states $|\alpha\rangle + e^{i\varphi_j}|-\alpha\rangle$ ($j = 1, ..., 8$) are prepared in certain phases φ as illustrated in the left.

where normalization factors are omitted for simplicity. Then one makes homodyne detection on one of the two modes. An ideal homodyne detection corresponds to a projection onto the quadrature eigenstate $|x\rangle$, and for the measurement outcome x, one obtains the conditional output state

$$\begin{aligned}&\langle x|(|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle)|0\rangle + \langle x|0\rangle(|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle)\\ &\propto \left(e^{-(x-2\alpha)^2/2} - e^{-(x+2\alpha)^2/2}\right)|0\rangle\\ &\quad + e^{-x^2/2}\left(|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle\right). \end{aligned} \tag{32}$$

Here, conditioned on the outcome $x = 0$, the first term vanishes and one obtains the amplified state $|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle$. More generally, the condition for the two inputs $|\alpha\rangle + e^{i\varphi_{1,2}}|-\alpha\rangle$ to be amplified is $\varphi_1 + \varphi_2 = \pi$. The amplified state has the phase $\varphi_1 - \varphi_2$ which implies that one can choose arbitrary φ at the output.

For further amplification, the process should be concatenated by carefully preparing the initial input states. Figure 5(b) is the schematic of the process to generate $|2\sqrt{2}\alpha\rangle - |-2\sqrt{2}\alpha\rangle$ by concatenating three amplification steps. The numbers represent the phase φ of each state. The same rule can be applied in a straightforward way for the iterative generation of a superposition of large coherent states with arbitrary φ.

Homodyne detection is a well matured technique and very high quantum efficiency ($\eta \geq 0.99$) has been achieved with current technology. It simplifies the experimental complexity, enhances the practical success probability compared to a use of two imperfect on/off detectors, and thus will increase the total feasibility of the experimental demonstration.

V. CONCLUSIONS

In this paper, a novel scheme for the conditional generation of a coherent state superposition with arbitrary ratio and phase has been proposed. The scheme uses a squeezed vacuum, beamsplitters, a coherent state ancilla, and two on/off detectors. We first showed that $c_+|\alpha\rangle + c_-|-\alpha\rangle$ for arbitrary $\{c_+, c_-\}$ can be approximated by an appropriate superposition of single- and two-photon subtracted squeezed vacua with very high fidelity ($F > 0.99$).

Such a superposition of photon subtracted states is conditionally generated by using ideal photon number resolving detectors and a qubit ancilla $b_0|0\rangle + b_1|1\rangle$. We have shown that this ideal scheme is also realized by the more practical scheme in which a displacement operation (coherent state ancilla) and on/off detectors are used. Even including realistic dark counts and low quantum efficiency (e.g. $\nu = 10^{-7}$ and $\eta = 0.1$), fidelities of more than 0.95 could be achieved with a highly transmissive beamsplitter ($T = 0.95$) and the squeezing of $r = 0.3$ (~ 2.6 dB), which corresponds to the amplitude of $\alpha = 0.95$ for the generated state. All these parameters are reasonably comparable with the recent experiments on single-photon subtraction [4, 5, 6].

We have also shown that our scheme is useful to simplify the setup of the conditional amplification of the coherent state superpositions originally proposed in [19, 20]. Our simplified version is quite feasible to demonstrate with current experimental techniques. Our scheme would also be useful to save the amount of required resources in other quantum information applications, including superposed coherent state based quantum computing [17, 18].

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[22] Note that the definition of $|\Psi_m^{(-)}\rangle$ in our paper is slightly different from the Dakna's original one [2] by a factor of $(-1)^m$. Here we also refer the photon number expansion of $|\Psi_m\rangle$ as [2]

$$|\Psi_m\rangle = \frac{1}{\sqrt{\mathcal{N}_m}} \sum_{n=0}^{\infty} a_{m,n}|n\rangle,$$

where

$$\begin{aligned} a_{m,n} &= \frac{(m+n)!}{\Gamma[(m+n)/2+1]\sqrt{n!}}, \\ &\times \frac{1+(-1)^{m+n}}{2}\left(\frac{\lambda T}{2}\right)^{(m+n)/2}, \end{aligned}$$

and

$$\mathcal{N}_m = \frac{1}{\sqrt{1-\lambda^2T^2}}\left[\frac{\lambda^2T^2}{1-\lambda^2T^2}\right]^m \times \sum_{k=0}^{[m/2]} \frac{(m!)^2}{(m-2k)!(k!)^2(2\lambda T)^{2k}}.$$

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